Quasilinear elliptic problems with a singular gradient term

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We consider the boundary value problem

$$\begin{cases} -\mathrm{div}(M(x)\nabla u) = \lambda u + m(x)\dfrac{|\nabla u|^q}{|u|^{q-1}} + f(x) & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

in a bounded domain $\Omega \subset \mathbb{R}^N$. Here M denotes an elliptic bounded matrix, $m \in L^\infty(\Omega)$, $1 < q < 2$ and the datum $f \in L^p(\Omega)$ for some $p > \frac{N}{2}$ is nontrivial and may change sign. We prove existence of solution for every $\lambda < \lambda^*$, the principal eigenvalue for the following problem

$$\begin{cases} -\mathrm{div}(M(x)\nabla u) = \lambda u + m(x)\dfrac{|\nabla u|^q}{u^{q-1}}\,, & \text{in } \Omega, \\ u > 0\,, & \text{in } \Omega, \\ u = 0\,, & \text{on } \partial\Omega. \end{cases}$$

References

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